<u>Defining Brushless DC Motors #2 of 4:</u>
<u>Relationship between Motor Sensitivity (Kt) & back emf (Kb)</u>

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In my first blog I presented an overview of a simple six-step procedure for defining a permanent magnet BDCM. In this second blog, I will show you how to establish the relationship between motor sensitivity (Kt) & back emf (Kb).

The expressions for torque and emf (electro-motive force) in a DC motor are [REF 1]

$$T = K\Phi I \qquad \text{EQU (1)}$$

and,

$$E = K\Phi\Omega \qquad \text{EQU (2)}$$

where:

T $= torque$
I $= armature\ current$
E $= EMF$
Ω $= angular\ velocity$
Φ $= air\ gap\ flux\ per\ pole$
P $= number\ of\ poles\ per\ revolution$
$N_s = number\ of\ conductors\ in\ series$

This can be determined from:

$N_s = \frac{z}{p^1}$ where z = number of conductors, and p^1 = number of parallel paths.

The DC current motor is an energy converter: i.e. electrical energy is converted to mechanical energy. The developed mechanical power is equal to the generated electrical power (neglecting friction) or

$$K\Phi = \frac{T}{I} = \frac{E}{\Omega} \qquad \text{EQU (3)}$$

The value of K can be derived as follows:

For N_s conductors in series, the average emf is: [REF1]

$$E = N_s \frac{\Delta\Phi}{\Delta t}\left[\frac{wb}{s}\right] = N_s\,\Phi\left[\frac{wb}{pole}\right]_P P\left[\frac{poles}{rev}\right]\frac{n}{60}\left[\frac{rev}{sec}\right] \qquad \text{EQU (4)}$$

Now substitute EQU (4) into EQU (2) to get:

$$(N_s\,\Phi p^n)\ /\ 60 = K\Phi\Omega$$

$$\therefore\ K = \frac{Ns\,p^n}{60\Omega} \qquad \text{EQU (5)}$$

Note that $\Omega = 2\Pi\ n/60$

$$\therefore K = \frac{Ns\ P}{2\Pi} \quad \text{a constant} \qquad \text{EQU (6)}$$

As a result we see that K is a constant determined by the <u>number of poles</u> and <u>conductors</u> in the motor.

For a permanent magnet DC motor (e.g. samarium cobalt) Φ, air gap flux per pole is constant since the field current and, thereby, flux density is constant.

Therefore, the expressions for torque and EMF in a <u>DC permanent magnet motor</u> are:

$$T = K_t I \qquad \text{EQU (7)}$$

and,

$$E = K_{b'}\ \Omega$$

$$\therefore\ E = K_b n$$

where $K_b = K_{b'}\ \frac{2\Pi}{60}$ EQU (8)

In EQUs (7) and (8):

$\frac{T}{I} = K_t$ = motor sensitivity constant, $\frac{in-oz}{amp}$

$\frac{E}{n} = K_b$ = back emf constant, $\frac{volts}{1000\,rpm}$

Substituting EQUs (7) and (8) into EQU (3) we see that

$K_t \;=\; cK_b$ EQU (9)

Where c is a conversion factor:

recall $K_t = \frac{T}{I} \;=\; \frac{in\text{−}oz}{Amp}$

and $K_b = \frac{E}{n} \;=\; \frac{volts}{Rpm}$

therefore $\frac{in\text{−}oz}{amp}$= c $\frac{volts}{Rpm}$

$$\frac{\frac{in-oz}{Amp}}{\frac{volts}{rpm}} = \frac{in-oz-Rev}{volt-min-Amp}$$

volt-sec-Amp (<u>min</u>) = newton-meter
(60 sec)

Since VI has units of $\frac{Joules}{sec}$

add 1 Joule = 1 Newton-meter by definition

$\therefore \frac{in\text{−}oz\text{−}Rev}{volt\text{−}min\text{−}Amp} = \frac{in\text{−}oz\text{−}Rev}{Newton\text{−}meter}\frac{min}{60\ sec}$

$$\frac{in-oz-Rev}{Newton-meter}\ \frac{min}{60\ sec}\ \frac{Newton}{.2248\ 1b}\ \frac{2\Pi\ Radians}{Rev}\ \frac{1b}{16oz}\ \frac{meter}{39.37in}$$

$\therefore \qquad \frac{in\text{−}oz\text{−}Rev}{Newton\text{−}meter}\ (.0007395) = \frac{in\text{−}lb}{in\text{−}lb} = 1$

$\frac{\frac{in\text{−}oz}{Amp}}{\frac{volts}{Rpm}}(.0007395) = 1$

$\frac{in\text{−}oz}{Amp}(.0007395) = \frac{volts}{Rpm}$

$\frac{in\text{−}oz}{Amp} = 1352\ \frac{volts}{Rpm}$

$\frac{in\text{-}oz}{Amp} = 1.352 \quad \frac{volts}{1000\ Rpm}$

More to come

We now have established the relationship between motor sensitivity (Kt) & back emf (Kb). In the upcoming blogs we will derive the constants and finally give an example.
Stay tuned!

Let's Talk

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